Measurement of reflective liquid crystal displays

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Cell gap thickness is one of the most important design parameters for liquid crystal displays (LCD). Existing measurement methods only apply to transmittive LC cells. These methods are obviously unsuitable for measuring the cell gaps of reflective LCDs with internal reflectors. This kind of LCD includes single polarizer reflective displays and liquid crystal on silicon displays. The objective of this article is to introduce various reflective methods of LCD cell characterization that are both applicable to transmittive and reflective LCDs.
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I. INTRODUCTION

Reflective liquid crystal displays (LCD) have been studied intensively in recent years. They are important for many applications such as in low power single polarizer displays,[1–3] or in liquid crystal on silicon (LCoS) microdisplays.[4,5] The former reflective display can be constructed on active matrix thin film transistor substrates as well as in passive matrix displays. For these reflective displays, cell gap control is as important as in conventional supertwisted nematic (STN) displays. Cell gap nonuniformity will lead to obvious visual defects and coloration.

For the case of LCoS microdisplays, the need for accurate cell gap measurement and control is also obvious, both in mass production quality assurance and in prototype development. However, all the previously reported methods for cell gap measurements are meant for transmissive cells.[6–13] These methods require the incident light to pass through the test cell and then the transmitted light is analyzed. They are not directly applicable to the measurement of reflective LC cells.

In this report, two methods for reflective display cell gap measurement are discussed. One of them relies on a spectral measurement similar to our earlier method for transmittive cell. In the second method, only a single wavelength is used. This second method can be easily implemented with a small laser. Unlike the transmittive case, the twist angle of the LC cell cannot be determined by these methods. However if a transmissive cell is measured by the reflective methods, the twist angle and cell gap can still be determined simultaneously as in Ref. 13. Principles of this single laser method have been reported briefly.[14] There are some recent reports on reflective cell measurement using a single wavelength as well.[15–17] They require the use of compensating films. Moreover, these methods rely on curve fitting to formulas to obtain the LC cell retardation. All these methods differ in ease of implementation and degree of accuracy.

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II. THEORY

Recently, we have reported a spectroscopic ellipsometric method for transmissive LC cell measurement.[13] In that method, we first derived the linear polarization output solutions, namely the LP1 and LP2 solutions, and then by applying these solutions, we were able to deduce the twist angle, twist sense, retardation, and rubbing directions for common twisted nematic and STN LCDs. The fundamental difference between the measurement of a transmissive and a reflective cell is that, the incident light of a reflective cell has to pass through the LC cell twice before it can be analyzed. This fact makes the optics of a reflective cell significantly different from that of a transmissive cell. Below we shall discuss the optics of such reflective systems by means of the optical equivalence theorem. In addition to the LP1 and LP2 solutions, we shall also make use of the CP (circular polarization output for single pass of the LC cell) solution that is particularly useful for reflective LC cell measurements.

The Jones matrix of a twisted nematic birefringence layer can be written as

$$M=\begin{pmatrix} a+ib & -c+id \\ c+id & a-ib \end{pmatrix} \tag{1}$$

which is a unimodular unitary matrix, where *a, b, c,* and *d* are real numbers represented by

$$\begin{aligned} a&=\cos\beta\cos\phi+\frac{\phi}{\beta}\sin\beta\sin\phi,\\ b&=-\frac{\delta}{\beta}\sin\beta\cos\phi,\\ c&=\cos\beta\sin\phi-\frac{\phi}{\beta}\sin\beta\cos\phi,\\ d&=-\frac{\delta}{\beta}\sin\beta\sin\phi, \end{aligned} \tag{2}$$

where $\beta=\sqrt{\delta^2+\phi^2}$, $\delta=\pi d\Delta n/\lambda$, ϕ is the twist angle, d is the cell gap, Δn is the birefringence, and λ is the incident light wavelength. The input director of the LC cell is assumed to be parallel to the *x*-axis. Here $(\phi,d\Delta n)$ are the

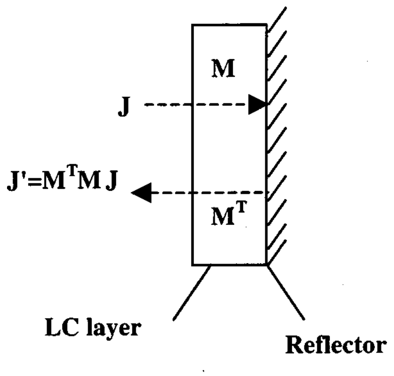

FIG. 1. A reflective LC cell with internal reflector. J and J' are the input and output Jones vector respectively.

liquid crystal cell design parameters. It is also known that for every unitary Jones matrix, there is an optically equivalent system consisting of one waveplate and one polarization rotator.[18] Thus one can have the following equation:

$$M(a,b,c,d)=R(\chi)WP(\Gamma,\alpha), \tag{3}$$

where $WP(\Gamma,\alpha)$ is the Jones matrix of a waveplate with retardation angle Γ and the slow axis at angle α to the x-axis, and $R(\chi)$ is the polarization rotator Jones matrix with rotation angle χ.

From Eq. (3), it is not difficult to get the following relations between the LC parameters ($\phi, d\Delta n$) and the equivalent model parameters (α, Γ, χ):

$$\tan 2\alpha=\frac{\phi}{\beta}\tan\beta, \tag{4}$$

$$\sin^2(\Gamma/2)=(\delta^2/\beta^2)\sin^2\beta, \tag{5}$$

$$\tan\chi=(\beta\sin\phi\cos\beta-\phi\cos\phi\sin\beta)/(\beta\cos\phi\cos\beta+\phi\sin\phi\sin\beta) \tag{6}$$

Consider a reflective twisted nematic LC cell as shown in Fig. 1. If the LC cell is represented by the equivalent optical system, then the Jones matrix of the entire reflective system is represented by

$$M^TM=WP(\Gamma,\alpha)R(-\chi)R(\chi)WP(\Gamma,\alpha)=WP(2\Gamma,\alpha), \tag{7}$$

where M^T is the transpose of M which represents the Jones matrix of the LC cell with light travels from the opposite direction.[19–21] It is therefore concluded that the reflective LC system is equivalent to just one retardation waveplate and that the resulting Jones matrix has no χ dependence. This is an interesting result that will have significant bearing on all reflective cell measurements.

In our previous report on the transmissive ellipsometry method, we used the LP1 solution such that the LC cell works as a polarization rotator to deduce the twist angle. In the LP1 condition, the polarization rotator angle χ in Eq. (3) is equal to the twist angle ϕ. Therefore from Eq. (7), we know that one cannot solve for the twist angle by using the LP1 solution any more in a reflective LC cell. If the twist angle of the LC cell is known, however, it is still possible to find the LP1, LP2 solutions, in addition to the CP solution by using Eq. (7), as detailed later.

For a linear polarized input light incident on a reflective cell, if one requires the output light to be linearly polarized as well, the final equivalent waveplate must be either a full-wave plate or a half-wave plate. Alternatively, the linearly polarized input light must be along the slow (or fast) axis of the equivalent waveplate. If the waveplate $WP(2\Gamma,\alpha)$ is a full-wave plate, then Eq. (5) is equal to either 1 or 0. For a nonzero twist angle this condition leads to the first linear output solution LP1:

$$\sin\beta=0 \quad \text{or} \quad \delta^2+\phi^2=(N\pi)^2. \tag{8}$$

If the waveplate $WP(2\Gamma,\alpha)$ is a half-wave plate, then $WP(\Gamma,\alpha)$ is a quarter-wave plate and Eqs. (5) gives

$$1/2=(\delta^2/\beta^2)\sin^2\beta. \tag{9}$$

If the reflected linear polarized light is to be orthogonal to the incident one, then the input polarized light must be at $\pm 45°$ to the waveplate axis, or the incident angle should be given by

$$\tan 2\alpha_{\mathrm{CP}}=\frac{-\beta}{\phi}\cot\beta. \tag{10}$$

Eq. (10) can be derived directly from Eq. (4). The subscript CP for α means that the light polarization state just before the reflector is circular. Eqs. (9) and (10) are the conditions for linear orthogonal output. They will henceforth be called the CP solutions.

Lastly, if the incident linear polarized light is along the azimuthal angle α of the equivalent waveplate, the reflected light will also be linearly polarized. The incident polarization angle α is given by Eq. (4). Eq. (4) is thus written as

$$\tan 2\alpha_{\mathrm{LP2}}=\frac{\phi}{\beta}\tan\beta. \tag{11}$$

This last linear polarization output solution is the LP2 solution. The subscript LP2 for the angle α means that when the incident polarization angle is at α_{LP2}, the reflected linear light is a LP2 type solution. Hereafter, the angle α without a subscript is a general incident light polarization angle.

Thus, the mathematical tools for the characterization of reflective LC cells are all in place. Imagine a simple experimental setup as depicted in Fig. 2. The light source can be a broadband light source or a single wavelength laser source; the detector is correspondingly a spectrophotometer or just a light power meter. Before we proceed to the experimental procedure, let us examine some properties of the three linear polarized reflection solutions.

(1) LC cells and alignment conditions satisfying LP1 or LP2 solution will give linear polarization light just before the reflector after a single pass through the LC cell. For LC cells satisfy the CP solution, the state of polarization before the reflector is circular.

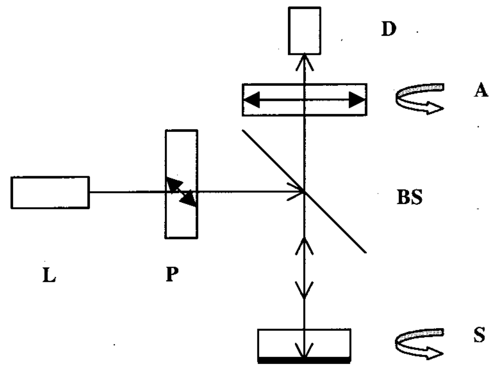

FIG. 2. Schematic diagram of the reflective measurement system. **L**-Light source, **P**-Polarizer, **BS**-Beam splitter, **A**-Analyzer, **D**-Detector (Spectrometer), **S**-Sample on a rotary stage.

(2) The reflected linear polarized light for LP1 and LP2 solutions are parallel to the incident one while for the CP solution the reflected light is orthogonal to the incident light.

(3) The LP1 solution given by Eq. (8) has no dependence on the input polarization angle α. That means the LP1 solution exists irrespective of the LC cell rotation angle, while the LP2 solution is not.

(4) Not all LC cells have LP1 and CP solutions. They are limited to those LC cells with parameters satisfying Eqs. (8) or (9). The solution bands of the LP1 and CP solution in the 400–800 nm range are given in Figs. 3 and 4. However, it is true that for every LC cell, there exists an incident angle α_{LP2} such that Eq. (11) holds. That means one can always find a LP2 solution for any incident light wavelength.

(5) Because of property (4), it is necessary to find the LP1 and CP solutions by an experimental setup with a broadband light source. To find the LP2 solution, a single wavelength laser source is enough.

(6) No matter which solution is obtained, once that condition is identified, the $d\Delta n$ value of the LC cell can be calculated from the experimental parameters.

III. ILLUSTRATIVE EXAMPLES

A. Reflective LCoS cell

A 52° twist LCoS cell was measured by the spectral method first. The LP1 and CP solution can be obtained by

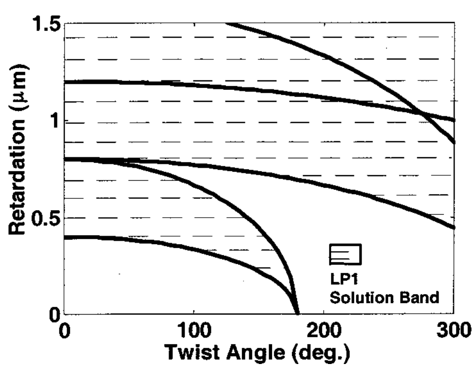

FIG. 3. Solution space of the LP1 solution in the 400–800 nm range.

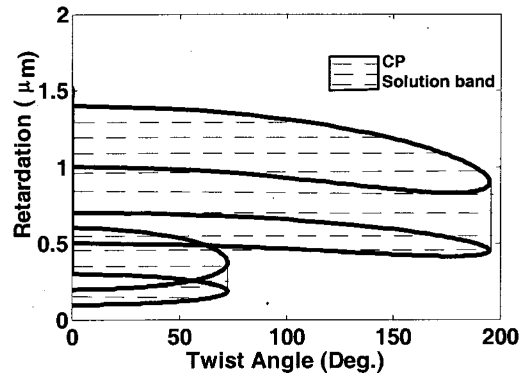

FIG. 4. Solution space of the CP solution in the 400–800 nm range.

simple procedures of identifying some special features of the reflected light, such as dependence on the rotation angle and null reflection. After each solution is identified, the angles of α and the null wavelength λ can be recorded and used to calculate $d\Delta n$ using Eq. (8) or (9).

The LP2 solution of the LCoS cell can be measured by the laser method. This time the light source was a helium neon laser and the detector was just a light power meter. The sample cell was put on the sample stage and then the stage was rotated to search for light reflection extinction. From Eq. (11), given ϕ and λ, the solution curve of $d\Delta n$ as a function of the measured α is plotted in Fig. 5. This is a useful diagram, as the $d\Delta n$ solution can be read out directly for any value of measured α. For the 52° LCoS cell in this experiment, the angle α was determined to be $-7°$.

Table I is a summary of the measurement results. These results are also plotted in Fig. 6. It is noted that a very useful by-product of this cell gap measurement is the dispersion of Δn. Since the different LP1, LP2, and CP solutions correspond to different wavelengths, we can obtain the dispersion of Δn. Figure 6 shows the strong wavelength dependence of Δn. It can be fitted by a single band dispersion model and the corresponding Cauchy formula[22]

$$d\Delta n(\lambda)=A+B/\lambda^2+C/\lambda^4 \tag{12}$$

where A, B and C are the Cauchy coefficients. The curve in

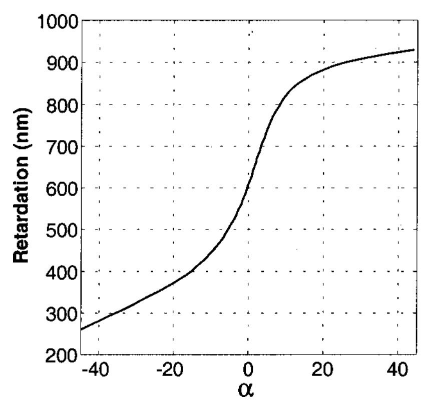

FIG. 5. LP2 solution curve of a 52° twist LC cell for a wavelength of 632.8 nm.

TABLE I. Experimental results for the 52° LCoS cell.

Solution type	Wavelength (nm)	Retardation (nm)
CP	450	551.5
LP1	530	507.1
LP2	633	476

TABLE II. Experimental results for a 240° STN cell measured by both the transmissive and reflective methods. $LP2_{45}$ is the LP2 solution obtained when α equals 45°. T_{50} is a solution obtained for 50% transmission when LP1 solution is satisfied and α equals 45° (see Ref. 13). T or R in bracket indicates transmissive or reflective methods.

Solution type	Wavelength (nm)	Retardation (nm)
$LP2_{45}$ (T)	464	981.3
T_{50} (T)	512	928
LP1 (T,R)	587,589	875,878
LP2 (R)	633	856 ($\alpha=-6.5°$)

Fig. 6 is a theoretical fit using Eq. (12). The Cauchy coefficients obtained for this LC correspond to the given values very well.

B. Transmissive STN cell

As mentioned above, the reflective method is also applicable to transmissive cells. An obvious method is to place a reflector behind the sample cell and then carry out the measurement procedure as described in Sec. III A in the same manner. Note that it makes no difference to the measurement results by placing either side of the LC cell toward the light source.

A sample 240° twist STN cell was measured by both the spectral and laser method. The measurement results are listed in Table II. In order to compare the reflective measurement results with the transmissive spectroscopic method results, we had also measured the same STN cell by the transmissive method as described in Ref. 13. The results are shown in Fig. 7. Details of the transmissive measurement method and definitions of the $LP1_{45}$ and T_{50} solutions can be found in Ref. 13. It can be seen that for the LP1 solution, which can be obtained by both methods, the results agree to within 1%. Note here that we do not have the CP solution for the STN cell because, from the CP solution band diagram (Fig. 4), no CP solution can be obtained beyond about 195° twist angle.

IV. DISCUSSION

We have demonstrated that by using a reflective setup, it is possible to measure the cell gap of both reflective and transmissive LCDs. In the LCoS cell example, we used a 52° twist cell with retardation of ~500 nm, therefore a CP and a LP1 solution is obtainable in the visible range. However if we are going to measure a mixed-mode TN (MTN)[23,24] cell with 90° twist angle and retardation of ~300 nm, we will not be able to get both the CP and LP1 solutions. Fortunately we still have the LP2 laser method of obtaining the retardation. Recall that we can always have a LP2 solution for any given wavelength, provided that the twist angle and the front surface rubbing direction are already known. In the discussions above, the measurement methods are divided into spectral and laser (or monochromatic) light method for convenience. In fact there needs to be just one spectroscopic measurement setup. It is because one can always perform the monochromatic light method by either concentrating at a particular wavelength in a spectrophotometer or by tuning the light source at a particular wavelength using a monochromator.

The reflective method can also be used to measure the twist angle and retardation of a LC cell with no internal reflector (or transmissive cell). This method involves the finding of the characteristic angle [Eq. (6)] and phase angle [Eq. (5)] of the equivalent system of waveplate and polarization rotator. Using a single wavelength laser source, one can first find the LP2 solution for the LC cell with either side faces up. Record the incident polarization angle as θ_1. This is the primary characteristic direction of the LC cell. If we turn over the LC cell and measure the corresponding LP2 solution again, we have a different characteristic direction θ_2. It is easy to prove that the angle between these two characteristic directions is the characteristic angle χ. Recall

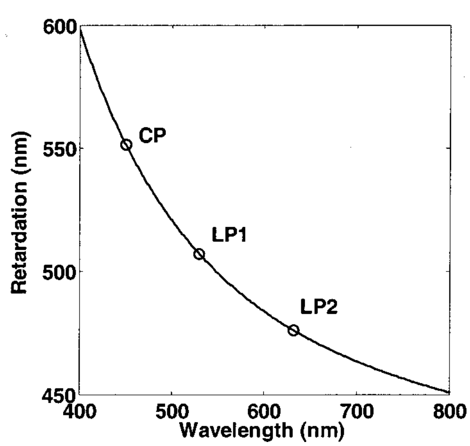

FIG. 6. Measurement results of the 52° LCoS cell. Curve through data points is a fitting using Cauchy type dispersion.

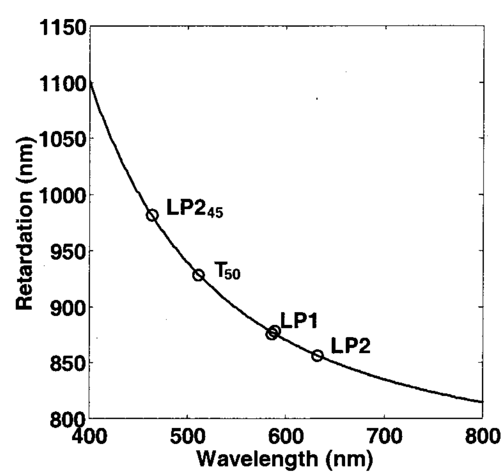

FIG. 7. Measurement results of a 240° STN cell by both the transmissive and reflective methods. Curve through data points is a fitting using Cauchy type dispersion.

that in Eq. (7), the LC equivalent model is simply a waveplate $WP(2\Gamma,\alpha)$. Therefore, by inserting a variable compensator with an axis perpendicular to α, the characteristic phase Γ can be determined. Once (Γ, α) is obtained, the $(\phi,d\Delta n)$ values can be obtained by solving Eqs. (5) and (6). Detailed discussions on this characteristic parameter method and associated techniques to find $(\phi,d\Delta n)$ will be published separately.[20]

V. CONCLUSIONS

In this article, we have presented two methods for the measurement of the cell gaps of reflective LC cells. The LP1, LP2, and CP conditions for the preservation of polarization in a LC cell are extensively used in these measurements. By either the spectroscopic or the single wavelength light method, the LC retardation can be obtained. The results are accurate and the methods are rather easy to implement. These methods should find extensive applications in the development and manufacturing of LCoS and other reflective LC cells.

Even though we have presented results on sample cells that are quite accurate, these methods can be further improved in principle. As seen in Fig. 5, the solution curve for the CP condition is rather steep. Thus any experimental measurement inaccuracy in the angle will translate to a significant experimental error in $d\Delta n$. Several methods can be used to overcome this difficulty and to improve the measurement accuracy. They involve the use of known retardation films for retardation compensation. These results will be discussed in some other publications.

ACKNOWLEDGMENT

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