44.3: A Liquid Crystal Lens with Non-uniform Anchoring Energy

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Abstract

The study of a tunable liquid crystal lens having a uniform cell gap, uniform applied voltage and non-uniform anchoring energy was done. Optimal parameters of such a lens were found and discussed. We demonstrated that producing a desired director gradient profile with non-uniform spatial distribution of the anchoring energy is a good alternative for other known methods applied for liquid crystal lenses.

1. Introduction

Many photonic devices require lenses with variable focal distance. A zoom lens system usually consists of a group of lenses, the separation distances between which are mechanically adjusted. Mechanical adjusting processes are, as a rule, complicated and bulky. That is the reason for developing a new type of tuneable lenses that are compact, lightweight, low-cost, and efficient. Such lenses are highly desirable and urgently needed for purposes of adaptive optics, optoelectronics, machine vision, stereo displays and eyeglasses applications. Due to the property to be controlled with an external electrical field, liquid crystals (LC), are especially attractive for tuneable lenses[1-7]. Combined with conventional optics, the liquid crystal lenses could provide adaptive focusing and scanning of optical beams for laser beam technology.

To obtain a switchable gradient profile within LC layer for LC lenses, several methods have been proposed [1-7]. All of them can be classified into three categories: 1) lenses having a non-uniform profile of the cell gap and uniform applied voltage; 2) lenses with a uniform cell gap and non-uniform spatial distribution of the applied voltage; and 3) lenses with non-uniform profile and non-uniform spatial distribution of the applied voltage. An LC lens, which we intent to consider in this paper, cannot be included in none of the mentioned above categories because it has a uniform cell gap and is controlled with a uniform electrical field. The needed gradient profile is achieved due to non-uniform surface conditions, in particular, a spatial distribution of the anchoring energy. We consider an LC lens composed of a LC layer sandwiched between two transparent flat substrates. Similar to the convention LC cells for displays, the inner sides of the substrates covered with transparent electrodes and alignment layers. Specific of the LC lenses to be considered is that the anchoring energy of the alignment layers is not a constant over the surface and is characterized with some spatial distribution. A non-uniform spatial distribution of the anchoring energy can be achieved with the use of photoalignment technique [8].

The goal of this work is to demonstrate abilities of LC lenses having non-uniform anchoring energy, as well as to give recommendations for developing practical applications.

2. Theoretical background

Simulation of LC lenses, like LC displays, includes two interrelated steps: finding the director distribution and calculation of the light propagation. To obtain the director distribution, we used numerical methods for minimization the Gibbs free energy per unit area of the system [8]:

$$G = \int \left(f_{el} + f_{diel} \right) dz + f_s \ , \tag{1}$$

where f_{el}, f_{diel}, f_s are the densities of free energy originating from the elastic deformation, dielectric interaction with the external field and the surface anchoring energy, respectively. The minimization of Eq.(1) corresponds to the case when the system is balanced with electric, elastic and surface forces. For the tilt-only deformation of a nematic LC, which we consider in the paper, the density of the elastic energy is:

$$f_{el} = \frac{1}{2}\left(K_{11}\cos^2\alpha(z) + K_{33}\sin^2\alpha(z) \right)\left(\frac{d\alpha(z)}{dz} \right)^2 , \tag{2}$$

where α is a tilt angle, the Z axis coincides with the normal to the LC cell surface, K_{11} and K_{22} are the spay and bend elastic constants, respectively.

The density of the dielectric interaction with the external electrical field $\vec{E}$ is:

$$f_{diel} = \frac{1}{2}\varepsilon_o \vec{E}\hat{\varepsilon}\vec{E} \ , \tag{3}$$

here $\hat{\varepsilon}$ denotes the dielectric tensor of LC material.

The anchoring energy density consists of two terms f_s^{top} and f_s^{bottom} that represent the anchoring energy density on the top and bottom boundaries. From Rapini-Papoular model and for an LC with symmetrical boundary conditions, we have [8]

$$f_s^{bottom} = f_s^{top} = \frac{1}{2}W\sin^2\left(\alpha - \alpha_o\right), \tag{4}$$

where W is the anchoring strength coefficient, α_o is the pretilt angle.

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The optical power D of an LC lens having a large radius R can be evaluated with Fresnel's formula [9]:

$$D = \frac{2\Delta\Phi\lambda}{R^2}, \tag{5}$$

where $\Delta\Phi$ is the phase difference between two extraordinary waves passed through the center and periphery of the lens, λ is the wavelength. The phase of an extraordinary wave is calculated as:

$$\Phi = \frac{2\pi}{\lambda}\int_L \frac{n_o n_e dr}{\sqrt{n_o^2 \sin^2\theta(r) + n_e^2\cos^2\theta(r)}}, \tag{6}$$

where L is the path along which the light ray propagates, n_o, n_e are the principal refractive indexes of the liquid crystal, θ denotes the angle between the wavevector and the director of the LC.

For rigorous calculations of the light propagation within the LC lens, we applied the eikonal approximation based on Fermat's principle[10,11]:

$$\frac{d}{ds}\left(n_{eff}(\vec{r})\frac{d\vec{r}}{ds}\right) = \vec{\nabla} n_{eff}(\vec{r}), \tag{7}$$

where $\vec{r}$ is the position vector on a point on the ray, ds is an element of the arc length along the ray, and $n_{eff}(\vec{r})$ is the effective refractive index for extraordinary wave:

$$n_{eff}(\vec{r}) = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\theta(\vec{r}) + n_e^2\cos^2\theta(\vec{r})}}$$

The propagation of the extraordinary wave was considered only because the direction of the Poynting vector of the extraordinary wave, in opposite to the ordinary wave, can be controlled with an external electric field. Solution of Eq.(7) satisfied to the following relation:

$$\left|\vec{\nabla}\vec{S}\right|^2 = \frac{n_o^2 n_e^2}{n_o^2 \sin^2\theta(\vec{r}) + n_e^2\cos^2\theta(\vec{r})} \tag{8}$$

In general, the direction of the Poynting vector of the extraordinary wave does not coincide with the wave normal. The angle between them (the dispersion angle) in the plane containing the crystal axis is

$$\tan\delta = \frac{(n_e^2 - n_o^2)\tan\theta}{n_e^2 + n_o^2\tan^2\theta} \tag{9}$$

To trace a ray, we used the algorithm described in Ref.[11]. It is based on iterations of the Eq. (7) for the wavevector ($\vec{k} = \frac{d\vec{r}}{ds}$) with the following determining the direction of the Poynting vector according Eq.(9). Refractions of the ray at the interfaces isotropic medium – anisotropic medium were carried according to coordinate-free approaches introduced by Fedorov [12,13].

Let a plane light wave falls on an LC lens at normal incidence. The wave passed through the LC lens is characterized with a wavevector $\vec{k}(x)$ that makes an angle γ with the normal to the LC lens, where the axis X is parallel to the lens surface (Fig.1).

The angle γ is a function of the spatial distribution of the anchoring energy *W(x)*, applied voltage V, cell gap d, as well as dielectric, elastic and optical constants of LC. For a certain LC, it is possible to write:

$$\gamma\big(W(x), V, d\big) = \arctan\left(\frac{x}{f}\right), \tag{10}$$

where f is the lens focal distance.

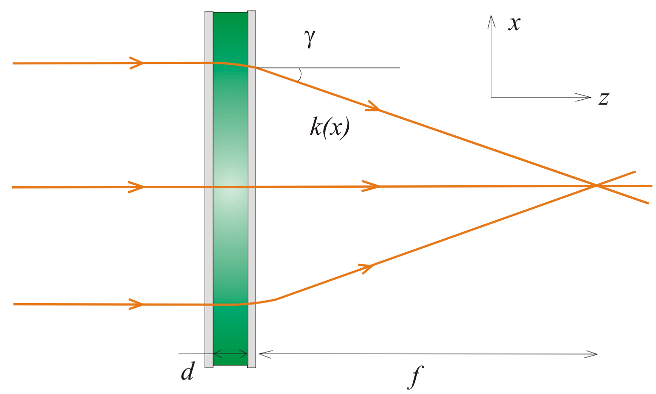

Fig.1 LC lens.

From Eq.10 follows that it is possible to find the spatial distribution of the anchoring energy *W(x)* when an LC cell acts as a lens for a fixed V, d and f. Changing V can invoke the changing of the focal distance. Our goal is to evaluate potential capabilities of such a tunable LC lens.

3. Results and discussions

According to Eq.(10), the focal distance or the optical power of the LC lens depends on the cell gap d and driving voltage V. Evidently, it must be some optimal values of d and V when effectiveness from the non-uniformity of the anchoring energy *W(x)* for the LC lens is maximal or, in other words, when the LC cell possesses maximal optical power for fixed d and V. Let us find these optimal values. As we mentioned earlier, the optical power of an LC lens is proportional to the phase differences $\Delta\Phi$ between two extraordinary waves passed through the center and periphery of the lens. In that places the distribution of the anchoring energies reaches its maximal and minimal values. Thus, it is reasonably to find such values of d and V when $\Delta\Phi$ is maximal. We considered *W(x)* changing within the range: from 5×10^{-6}J/m^2 (the weak anchoring energy) to 3×10^{-4}J/m^2 (the strong anchoring energy).

To be specific, we carried out the calculations of an LC lens filled with LC material E7 from Merck (k_{11}=12x10^{-12}N; k_{33}=19,5x10^{-12}N; $\varepsilon_{\perp}$=5,1; ε_{II}=19,6; n_o=1.5237, n_e=1.75). This material is good for LC lenses, because of high birefringence.

The calculated phase difference $\Delta\Phi$ as a function of the cell gap and applied voltage is plotted in Fig.2.

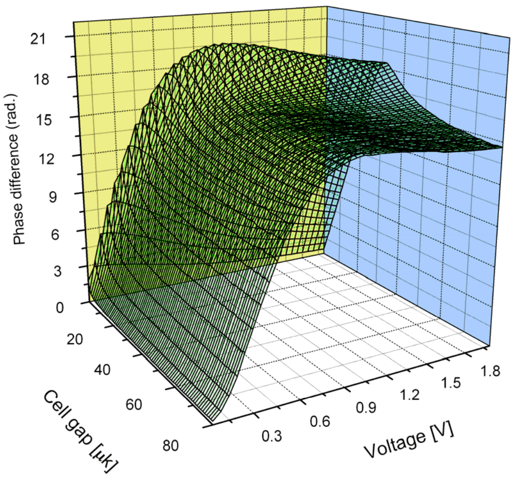

Fig.2. Phase difference between two extraordinary waves passed through the LC cell in places having the strong ($3x10^{-4}J/m^2$) and weak ($5x10^{-6}J/m^2$) anchoring energies versus the cell gap and applied voltage.

The graph in Fig.2 shows that the phase difference $\Delta\Phi$ does not change critically versus the cell gap, when the last more then 10μm and versus the voltage in the range 0.8 - 2V. The phase difference $\Delta\Phi$ reaches the zone with the maximal value when the cell gap is around 15μm and voltage is around 1V.

The further investigation was carried out for the LC cell with the cell gap *d*=15μm. We fixed the value of the weak anchoring energy at W=$5x10^{-6}J/m^2$ and varied the value of the strong anchoring energy. The dependence of the phase difference $\Delta\Phi$ versus applied voltage and the logarithm of the varied anchoring energy W is plotted in Fig.3. This dependence enables one to evaluate the needed distribution of the anchoring energy according to Eq.5. The smooth behaviour of the phase difference versus applied voltage makes possible to change the optical power of the LC lens in the wide range without essential distortions and defocusing.

Solving numerically Eq.(10) for a microlens with diameter 1mm and optical power 18 dioptres, we obtained the spatial distribution of the anchoring energy *W(x)*. Fig.4 contains results for both positive and negative lenses. The calculations of the director distributions and light propagations inside the LC lens that are necessary for solving Eq.(7) were done with Software developed by us for simulation of optical and electro-optical properties of LC devices.

The distribution presented in Fig.4 can be realised with photoalignment technique. The spatial distribution of the anchoring energy is achieved with the spatial distribution of energy of the illuminating light during the photo-polymerization.

Here we should note that not too many photoalignment materials posses property to keep controlled anchoring energy in the wide range from $5x10^{-6}$ J/m^2 to $3x10^{-4}$ J/m^2 [14]. However, from the graph presented in Fig.3, one can see that the phase difference $\Delta\Phi$ changes critically when logarithm of the anchoring energy varies from -12.2 to -10, that corresponds to W=$5x10^{-6}$ J/m^2 and $5x10^{-5}$ J/m^2, respectively. This new range is more tolerable for practical applications. In fact, it is the same as the range of changing of the azimuth anchoring energy.

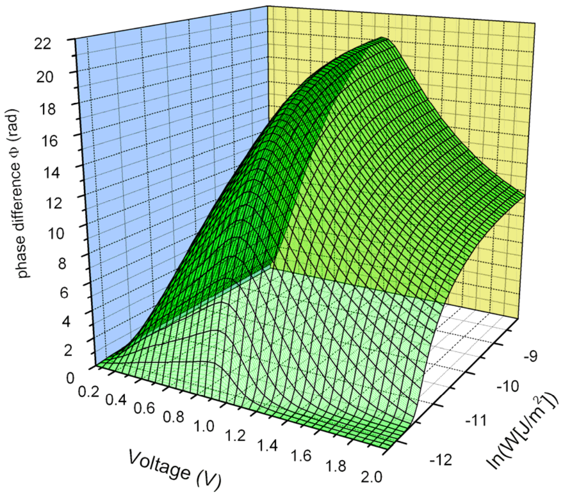

Fig.3. Phase difference between two extraordinary waves passed through the LC cell in places with the weak anchoring energy ($5x10^{-6}J/m^2$) and the varied one versus voltage and logarithm of the varied anchoring energy.

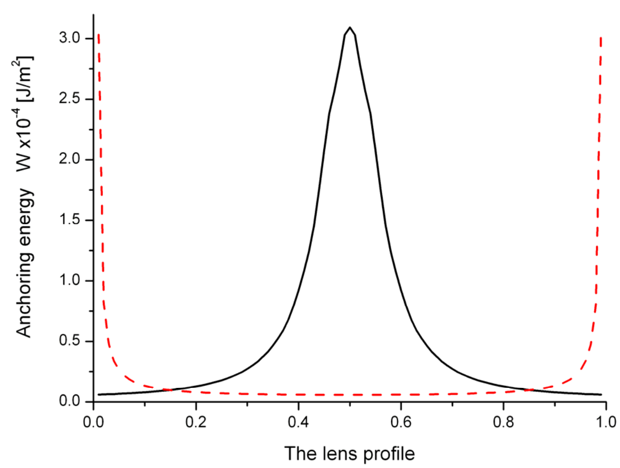

Fig. 4. Distributions of the anchoring energy for the positive LC lens (the solid curve) and for the negative LC lens (the dashed curve).

Fig. 5 shows the map of the director orientations for a positive LC lens having the spatial distribution of the anchoring energy W(x) as plotted in Fig.4 and applied voltage 1V.

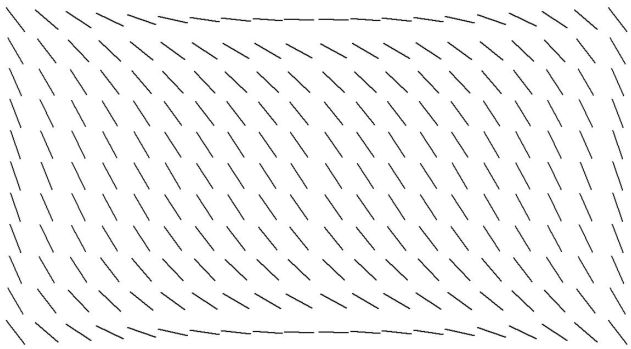

Fig.5. Map of the director distribution for the positive LC lens under applied voltage 1V.

4. Conclusion

We studied a tuneable LC lens having a uniform cell gap and voltage distribution but non-uniform spatial distribution of the anchoring energy. The method considered by us can be combined with any other known methods for reducing driving voltage of a LC lens or for simplifying the lens profile.

5. Acknowledgements

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6. References

[1] H. Ren, Y. H. Fan, S. T. Wu, Appl. Phys. Lett. **83**, 1515 (2003)

[2] H. Ren, S. T. Wu, Appl. Phys. Lett. **81**, 3537 (2002)

[3] H. Ren, S. T. Wu, Appl. Phys. Lett. **82**, 22 (2003)

[4] V. V. Presnyakov, K. E. Asatryan, T. V. Galstian,A. Tork, Opt. Express **10**, 865 (2002)

[5] B. Wang, M. Ye and S. Sato: IEEE Photonics Technol. Lett. **18**,79 (2006)

[6] B WANG, M YE and S Sato , J J Appl. Phys., **45**, 7813 (2006).

[7] H. Ren, Y.-H. Fan, S. Gauza, S.-T Wu, J. J. Appl. Phys **43**, pp. 652 (2004)

[8] V. G. Chigrinov, Liquid Crystal Devices: Physics and Applications (Artech House, Norwood, MA, 1999)

[9] D.C. O'Shea, T.J. Suleski, A.D Kathman, D.W. Prather Diffractive Optics, (SPIE Press, Washington 2004)

[10] M. Born and E. Wolf, Principles of Optics, 7th ed. (Cambridge U. Press, 1999).

[11] C. Jenkins, R. Bingham, K. Moore, G. D. Love, J. Opt. Soc. Am. A, **24**, 2089 (2007)

[12] F.I. Fedorov and F.I. Filippov Reflection and refraction of light in transparent crystals (in Rus.), (Nauka i technika, Minsk, 1976).

[13] F.I. Fedorov Theory of girotropy (in Rus.), (Nauka i technika, Minsk, 1976).

[14] V. Chigrinov, A. Muravski, H.S. Kwok, Phys. Rev. E, 68, 061702, (2003)