Extremely broadband and wide-angle retardation films

H. S. Kwok (SID Fellow)
X. J. Yu

Abstract — The design and construction of retardation films with any desired dispersion properties is reported. The method is simple and requires only conventional uniaxial retardation films. As an example, the design of retardation films which have constant retardation over the entire visible spectrum is demonstrated. The design methodology will be given. Specific design examples for broadband achromatic quarter-wave and half-wave retardation films are disclosed. These films show almost no wavelength dependence even at large viewing angles.

Keywords — *Retardation film, dispersion, viewing angles.*

DOI # 10.1889/JSID18.9.635

1 Introduction

Retardation films are used widely in optical systems. They can be biaxial or uniaxial. For wave propagating in the normal direction, the optical retardation for waves polarized in the principle direction is different from the optical retardation for waves polarized in the orthogonal direction, thus resulting in the changing of the polarization state of any input wave. The retardation value Γ of a retardation film is defined as the phase difference between the two orthogonal polarizations and is given by

$$\Gamma = \frac{2\pi d \Delta n}{\lambda}, \tag{1}$$

where d is the thickness and Δn is the birefringence of the retardation film, and λ is the wavelength of the input light. If $\Gamma = \pi$, then the retardation plate is a half-wave plate (HWP). If $\Gamma = \pi/2$, it is a quarter-wave plate (QWP).

Retardation films such as HWP and QWP have many applications such as in the polarization manipulation and phase compensation. In display engineering, they are used, for example, in viewing-angle enhancement and for dispersion compensation.[1] In projection systems, QWP and HWP are used in polarization conversion optics and in skew-ray compensation.[2,3] In all applications, the HWP and QWP should be constant in the entire visible spectrum (400–700 nm). However, conventional HWP and QWP using uniaxial retardation films have strong wavelength dependence. As well, their angular dependence is not totally desirable.

Various methods have been proposed to extend the wavelength range of retardation films.[4–9] Ishinabe *et al.* proposed the use of three or more biaxial films in a stack. Several inventions have also been proposed, making use of new retardation materials.[8,9] In this paper, we disclose a new concept for broadband film design, making use only of simple conventional wavelength dispersive films. Our method is based on stacking of ordinary dispersive retardation films to make the broadband retardation film. By using this method, general solutions for optimal designs can be obtained. Based on this new technique, specific examples of very broadband QWP and HWP can be obtained using commercial uniaxial retardation films. Most importantly, these QWP and HWP films show negligible wavelength dependence even at large viewing angles. This method can also be extended to create retardation films with any targeted dispersion properties. For example, it can have a dispersion that matches that of the birefringence of the liquid-crystal material. Thus, full compensation can be achieved for all wavelengths.

2 Design of broadband retardation films

In our method, the design of the retardation film such as the HWP and QWP is treated similar to that of the design of polarization interference filters (PIF).[5] PIF is a filter that rotates the polarization of a particular wavelength band by 90°, while leaving all the other complementary wavelengths unchanged. This task is accomplished by using a stack of retardation films. A PIF, together with an output polarizer, will therefore transmit a particular band of wavelengths. PIF is different from a conventional interference filter in that the two complementary spectra can be separated by a polarizing beamsplitter.[3,6] Now, a HWP rotates the polarization of incoming light by 90° if the c axis of the waveplate makes an angle of 45° with the incoming polarization. Thus, a broadband HWP can be regarded as equivalent to PIF with a very broad spectrum covering the entire visible range. Similarly, a QWP can be regarded as a special PIF which upon reflection rotates the polarization of the entire visible spectrum by 90°.

The basic structure of a PIF is shown in Fig. 1. It consists of a number of birefringent films placed between two polarizers. In the design of a PIF, the variables are the individual angles ϕ_i, as well as the retardation value Γ of the films. In most cases, we use only films with the same retar-

---

Received 01/17/10; accepted 07/14/10.

The authors are with The Hong Kong University of Science and Technology (HUST), Department of Electrical & Electronic Engineering, Rm. 2534, Clear Water Bay, Kowloon, Hong Kong; telephone +852-2358-7056, e-mail: eekwok@ust.hk.

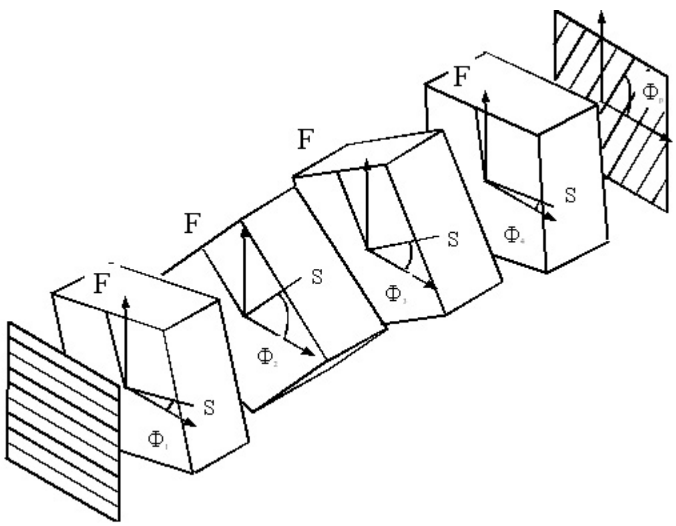

FIGURE 1 — The basic structure of PIF. F is the fast axis and S is the slow axis of the individual uniaxial retardation film. The input polarizer direction is defined as the *x* axis.

dation in order to simplify the manufacturing process. It will be seen that this is not a severe constraint on PIF design.

The transfer function $C(\omega)$ of an N component PIF system is given by[6]

$$C(\omega) = C_0 + C_1 e^{-i\tau_d\omega} + C_2 e^{-i2\tau_d\omega} \ldots + C_N e^{-iN\tau_d\omega}, \tag{2}$$

where $\tau_d = d\Delta n/c$, Δn is the dispersive birefringence of the retardation plate, d is the film thickness, and c is the velocity of the light in vacuum. Since $\omega = 2\pi c/\lambda$, thus

$$\omega\tau_d = \frac{2\pi\Delta n d}{\lambda} = \Gamma. \tag{3}$$

Equation (2) can therefore can be rewritten as

$$C(\Gamma) = C_0 + C_1 e^{-i\Gamma} + C_2 e^{-i2\Gamma} \ldots C_N e^{-iN\Gamma}. \tag{4}$$

It is well known that the Jones matrix can describe exactly the polarization state of light for normal incidence. Without loss of generality, we can define the x axis as the direction of the input polarizer. Then the Jones vector of the input light is

$$E_{in} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{5}$$

The Jones matrix of the i-th retardation plate is given by

$$\begin{aligned} W_i = R(-\phi_i) W_0 R(\phi_i) = e^{-i\psi} &\begin{bmatrix} \cos\phi_i & -\sin\phi_i \\ \sin\phi_i & \cos\phi_i \end{bmatrix} \begin{bmatrix} e^{-i\Gamma} & 0 \\ 0 & 1 \end{bmatrix} \\ \times &\begin{bmatrix} \cos\phi_i & \sin\phi_i \\ -\sin\phi_i & \cos\phi_i \end{bmatrix}, \end{aligned} \tag{6}$$

where ϕ_i is the optical axis orientation of the i-th retardation plate relative to the x axis, $R(\phi)$ is the polarization rotation matrix, and $\psi = \pi(n_e + n_o)(d/\lambda)$. The constant phase factor can be ignored in general without affecting the results because it does not affect the polarization. The Jones matrix of the combination of N waveplates is therefore given by

$$W = \prod_{i=1}^{N} W_i. \tag{7}$$

If we now rotate the axis so that the new x axis is in the direction of the output analyzer axis, the new output Jones vector will be given by

$$E_{out} = \begin{bmatrix} E_u \\ E_v \end{bmatrix} = \begin{bmatrix} \cos\phi_p & \sin\phi_p \\ -\sin\phi_p & \cos\phi_p \end{bmatrix} \prod_{i=1}^{N} W_i \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{8}$$

where ϕ_p is the angle of the output analyzer relative to the original x axis. Thus, E_u is the real output in frequency domain and E_v is the complementary function of E_u. It is easy to see that Eq. (8) can be written as

$$\begin{aligned} E_{out} = &\begin{bmatrix} \cos\phi_p e^{-i\Gamma} & \sin\phi_p \\ -\sin\phi_p e^{-i\Gamma} & \cos\phi_p \end{bmatrix} \\ &\times \prod_{i=2}^{N} \begin{bmatrix} \cos\theta_i e^{-i\Gamma} & \sin\theta_i \\ -\sin\theta_i e^{-i\Gamma} & \cos\theta_i \end{bmatrix} \begin{bmatrix} \cos\theta_1 \\ -\sin\theta_1 \end{bmatrix}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \theta_1 &= \phi_1 \\ \theta_2 &= \phi_2 - \phi_1 \\ &\vdots \\ \theta_N &= \phi_N - \phi_{N-1} \\ \theta_p &= \phi_p - \phi_N. \end{aligned} \tag{10}$$

Equation (9) can be expanded to give

$$E_u(\Gamma) = E_0 + E_1 e^{-i\Gamma} + E_2 e^{-i2\Gamma} \ldots + E_N e^{-iN\Gamma}, \tag{11}$$

where E_i is a function of relative angle θ_i of the birefringent films and polarizers. The actual output $E_u(\Gamma)$ given in Eq. (11) and the desired output $C(\Gamma)$ in Eq. (4) have the same format.

The idea of designing PIF based on the Jones matrix is the same as finding the values of θ_i such that

$$E_u(\Gamma) = C(\Gamma). \tag{12}$$

In the numerical procedure, the desired output $C(\Gamma)$ is given first, thus the coefficients C_i in Eq. (4) can be calculated. Comparing the coefficients in Eqs. (11) and (4), the relative angle θ_i of each birefringent film can therefore be obtained. The target output can be a broadband QWP or HWP. In addition, it can be a QWP or HWP with any targeted spectral properties. It can have a retardation that matches that of a liquid crystal for instance.

Let us also comment on the behavior of the dispersion of Δn. While the expansion in Eq. (11) is always correct, straightly speaking, the expansion in Eq. (2) is only valid if τ_d is a constant. Here, we assume the wavelength dependence of τ_d to be very small so that it does not have to be included in the harmonic expansion. However, as can be

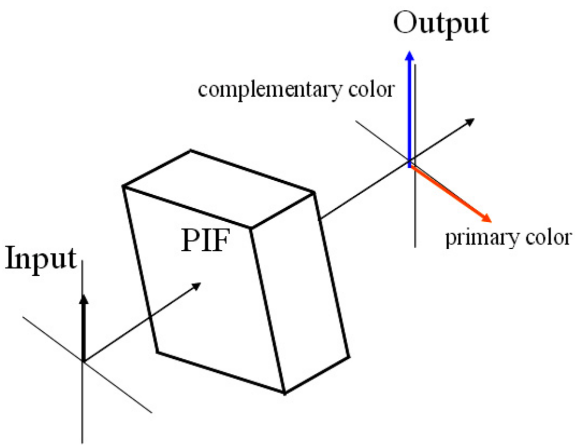

FIGURE 2 — The principle of PIF-type retardation film. The output can either have the designed spectrum if the output polarizer is along the *x* axis, or the complementary spectrum if the output polarizer is along the *y* axis.

seen in the specific examples of retardation-film designs, this restriction is rather mild and does not affect the broadband behavior of the invented retardation films. Thus, based on the algorithm given above, we have a new method for the design of any broadband retardation films using just conventional films.

3 Broadband HWP

The broadband HWP works in the transmissive mode, and rotates the input linearly polarized light by 90° for the entire visible spectrum as shown in Fig. 2. Thus, a HWP PIF between two crossed polarizers should have 100% efficiency. The desired transmission is given by

$$T(\Gamma)=|C(\Gamma)|^2=C(\Gamma)\bullet C^*(\Gamma)=100\%\,(400-700\ \text{mm}). \quad (13)$$

Because the constraint condition [Eq. (13)] is rather loose, there are multi-solutions to $C(\Gamma)$. This is different from PIF color filters where there are more constraints.[10,11]

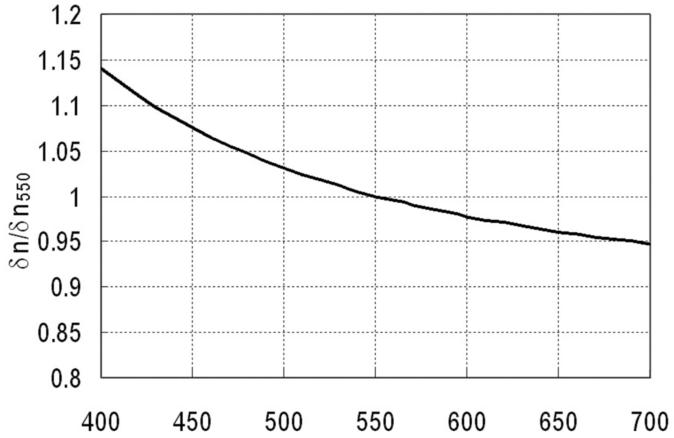

FIGURE 3 — The normalized dispersion property of the commercial film used in the experiments.

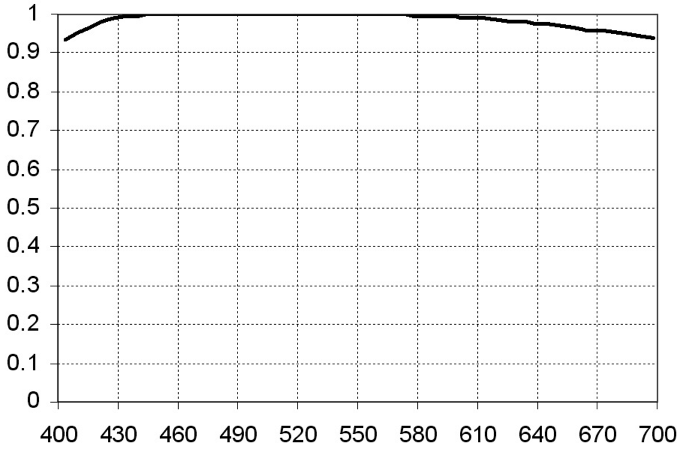

FIGURE 4 — Measured transmission of the two-layer broadband HWP between two crossed polarizers.

For easy fabrication, the broadband HWP or QWP should comprise three or less number of ordinary dispersive films. For a specific design of the broadband HWP, a commercial uniaxial HWP retardation film from Nitto–Denko with a retardation value $\Gamma = \pi$ at 540 nm is used. The normalized dispersion property of this film is given by the Cauchy's equation:

$$\Delta nd = 270\left(A_0+\frac{B_0}{\lambda^2}+\frac{C_0}{\lambda^4}\right), \quad (14)$$

where $A_0 = 0.8646$, $B_0 = 3.7018 \times 10^4$ (nm^2), and $C_0 = 1.2 \times 10^9$ (nm^4). The dispersion of Γ is shown in Fig. 3.

A two-layer broadband HWP has been so fabricated. Figure 4 shows the experimental transmission of this two-layer broadband HWP between two crossed polarizers. It can be seen that it practically has no wavelength dispersion over the entire visible range as designed.

A three-film design has also been fabricated using the same commercial films. The measured transmission of this three-layer broadband HWP between two crossed polarizers is shown in Fig. 5. The transmission data for a single commercial film is also shown in the same figure. It can be

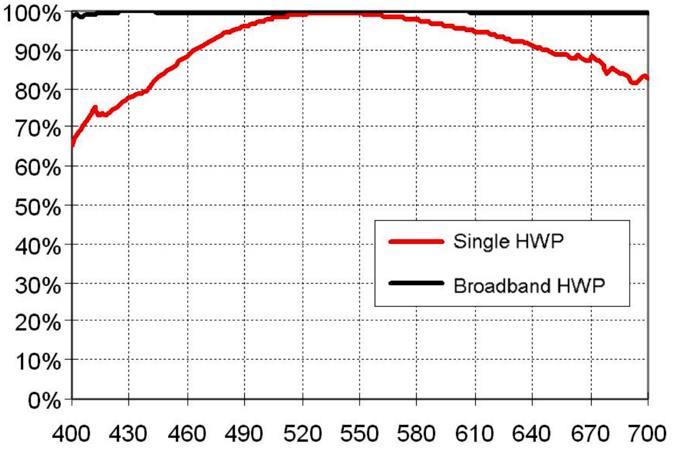

FIGURE 5 — Measured transmission of the three-layer broadband HWP between two crossed polarizers.

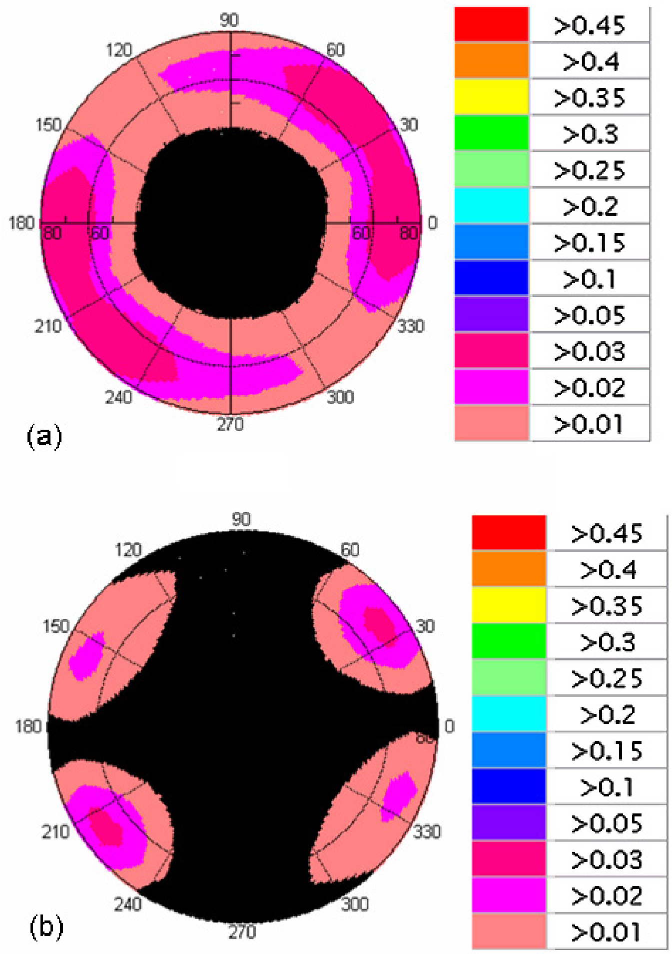

FIGURE 6 — Calculated transmission polar plots of the two-layer (top) and three-layer (bottom) design with parallel polarizers. Dark area indicates transmission of less than 0.01.

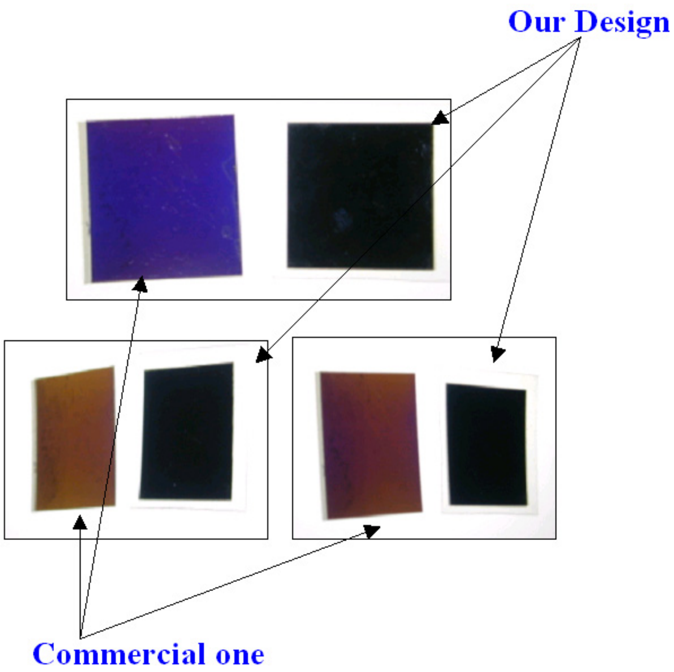

FIGURE 7 — Comparisons of the off-axis behavior between the broadband three-layer HWP and the commercial HWP.

seen that the three-layer film is better than the two-layer design and has no discernable change in transmission over the entire visible spectrum. This implies that the retardation is indeed π over the entire range and Eq. (13) is obeyed.

It turns out that the angular dependence of the retardation of the new HWP is also very good. Figure 6 shows the calculated polar plots of the transmission of the two- and three-layer BB HWP between parallel polarizers. It can be seen that the maximum transmission is only 1% for most of the viewing cone. For the three-layer design, the viewing cone is less than 1% leakage for the entire viewing cone.

Experimentally, the new broadband HWP and the commercial HWP can be more dramatically illustrated by placing them between two crossed polarizers and comparing them side by side. Figure 7 shows the picture. It can be seen that our HWP works very well for the entire visible spectrum even for larger viewing angles. Actually, it is not surprising that the three-layer HWP has better viewing-angle properties than the single-layer HWP. Since there are three retardation films with varying angles, the angular dependence tends to average out in the three-layer design. Indeed, the three-layer film behaves somewhat as a biaxial film.

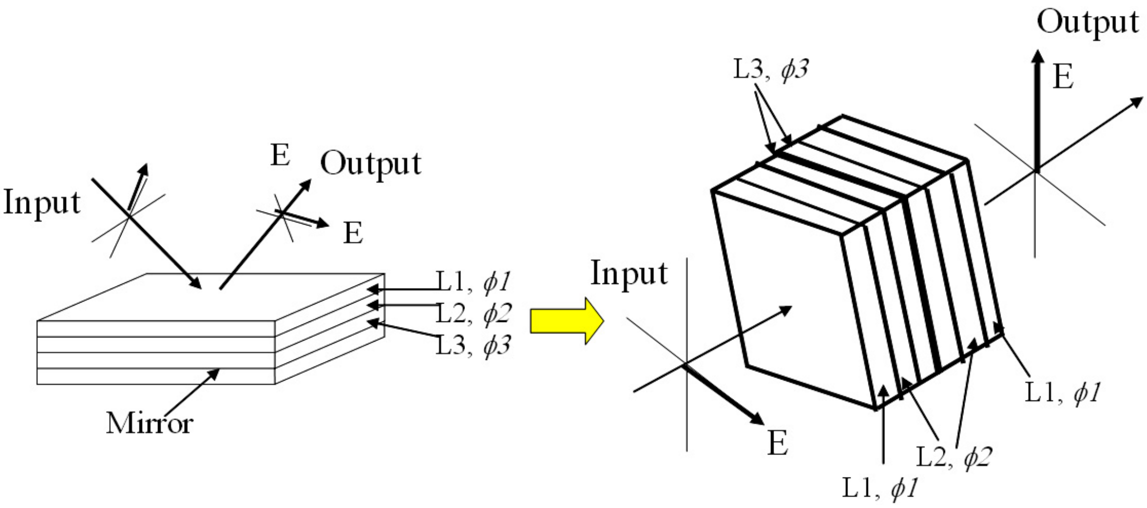

FIGURE 8 — The reflective PIF system and its equivalent transmissive PIF system.

4 Broadband QWP

A broadband QWP changes the linearly polarized input light into circular polarization if the input polarization is at 45° to its optical axis. In this case, we cannot simply repeat the formulation of Eqs. (7)–(10). However, we can allow the QWP to work in the reflective mode as shown in Fig. 7. The QWP will rotate the linearly polarized light by 90° upon reflection. This reflective PIF system can be equivalent to the transmissive PIF with two QWPs in a symmetric configuration, as shown in Fig. 8. In this equivalent HWP, there are twice the number of films with $\theta_1 = -\theta_p \pm 90°$, $\theta_2 = -\theta_N$, $\theta_3 = -\theta_{N-1}, \ldots$. Due to the symmetric configuration, the C_i in Eq. (3) should satisfy the relations[5]

$$C_0 = -C_N, C_1 = -C_{N-1}, C_2 = -C_{N-2}, \ldots \qquad (15)$$

To demonstrate this design, we use commercial HWP and QWP retardation films to make the broadband QWP. The retardation values of the conventional dispersive films are 270 and 140 nm at a wavelength of 540 nm, respectively:

$$\begin{aligned} \Delta nd &= 270\left(A_0 + \frac{B_0}{\lambda^2} + \frac{C_0}{\lambda^4}\right), \\ \Delta nd &= 140\left(A_0 + \frac{B_0}{\lambda^2} + \frac{C_0}{\lambda^4}\right). \end{aligned} \qquad (16)$$

It turns out that for the broadband QWP, a single commercial HWP and a single commercial QWP is sufficient to achieve excellent results. Such a two-film design is fabricated, and Fig. 9 shows the measured reflectivity comparison between the commercial QWP and our broadband QWP. The reflectivity is measured using crossed polarizers + QWP + mirror setup. It can be seen that indeed the broadband QWP changes the input linearly polarized light to output circularly polarized light for the entire visible region.

The calculated angular dependence of the retardation for both the two- and three-layer design are shown in Fig. 10. It shows the transmission of the two BB QWPs between parallel polarizers. It can be seen that the leakage is less than 5% for a large part of the viewing cone for the two-layer design and less than 3% for the three-layer design. In both cases, some serious leakage occurs at $\phi > 60°$ in some directions.

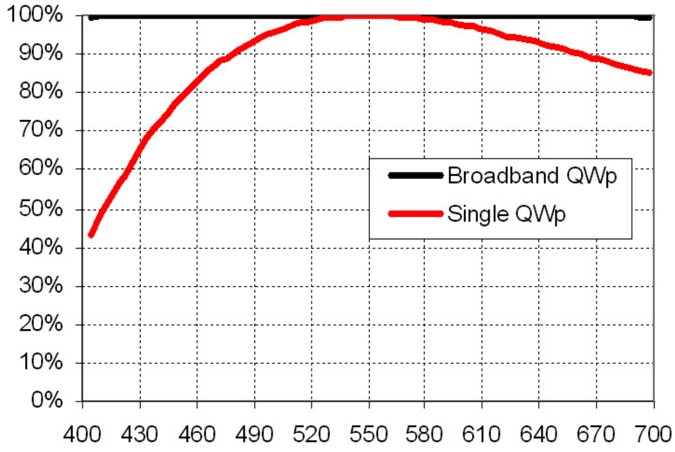

FIGURE 9 — Comparison between the commercial QWP and broadband QWP.

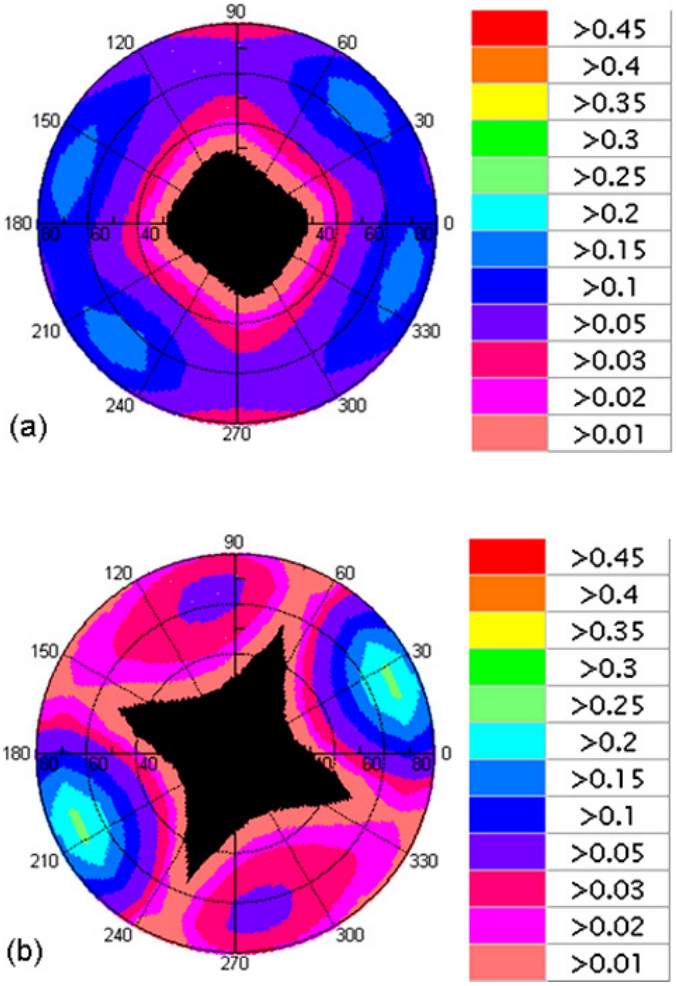

FIGURE 10 — Calculated transmission polar plots of the two- and three-layer designs with parallel polarizers and two QWP inside. The dark area indicates a transmission of less than 0.01.

Figure 11 shows the calculated real and imaginary parts of the y-component of the output Jones vector for the commercial QWP and the new two-layer broadband QWP

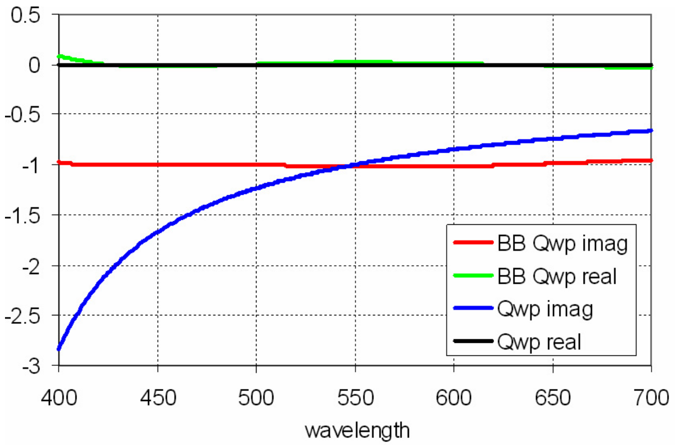

FIGURE 11 — Calculated real and imaginary parts of the y component of the output Jones vector of the two-layer QWP.

TABLE 1 — Summary of design results for some HWPs and QWPs.

Orientation	1st	2nd	3rd
HWP	20° (H)	64° (H)	
HWP	78° (H)	49° (H)	16° (H)
QWP	7° (H)	35° (H)	−79° (Q)

with the condition that the x-component of the output Jones vector is normalized and the input is linearly polarized light. It can be seen that the output of the commercial QWP is elliptically polarized, and the ellipticity is a function of the wavelength; while the output of our broadband QWP is $\begin{pmatrix}1\\-i\end{pmatrix}$ or $\begin{pmatrix}1\\i\end{pmatrix}$ and is independent of wavelength. It indicates that the output is circularly polarized for the entire visible spectrum.

5 Summary

In summary, we have introduced a new concept in the design of extremely broadband retardation films. The idea is to treat the retardation films as a polarization interference filter. The concept is general and can be applied to any retardation values. As well, only a minimum number of commercial films are needed in any design. So, the method has good commercial applications. In particular, we have applied this technique to HWP and QWP consisting of a stack of two or three conventional dispersive retardation films. The optical properties of these stacks are extremely desirable for many optical applications because there is practically no dispersion and the acceptance angle is very large. Table 1 summarizes the results for some designs of HWP and QWP.

As mentioned, it is also possible to tailor the dispersion properties of any retardation for exact wavelength compensation. For example, one can design QWP or HWP with retardation values that match exactly the birefringence of a liquid-crystal material for all wavelengths. This will allow exact compensation in many cases.

Acknowledgments

The authors thank Mr. Jacob Ho for assistance in making the samples. This research was supported by the Hong Kong Government Research Grants Council Grant Number 614408.

References

1 P. Yeh and C. Gu, *Optics of Liquid Crystal Displays* (Wiley Inter-Science, 1999).
2 W. C. Yip *et al.*, *Appl. Opt.* **36**, 6453 (1997).
3 M. Robinson *et al.*, *Polarization Engineering for LCD Projection* (Wiley, 2005).
4 S. Pancharatnam, *Proc. Indian Acad. Sci.* **A41**, 130 (1955).
5 T. Ishinabe *et al.*, *SID Symposium Digest* **32**, 906 (2001).
6 Z. Ge *et al.*, *IEEE J. Display Technol.* **4**, 129–138 (2008).
7 Q. Hong *et al.*, *Appl. Phys. Lett.* **86**, 121107 (2005).
8 X. Zhu and S-T. Wu, “Broadband quarter-wave film device including in combination a chromatic half-wave film and a TN-LC polymeric film,” U.S. Patent No. 6,922,221.
9 M. Verrall *et al.*, “Combination of optical elements,” U.S. Patent No. 6544605.
10 X. J. Yu *et al.*, *Displays* **23**, 145 (2002).
11 E. O. Ammann, *J. Opt. Soc. Am.* **56**, 943 (1966).